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Energy barrier to decoherence

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We propose a ground-state approach to realizing quantum computers. This scheme is time-independent and inherently defends against decoherence by possessing an energy barrier to excitation. We prove that our time-independent qubits can perform the same algorithms as their time-dependent counterparts. Advantages and disadvantages of the time-independent approach are described. A model involving quantum dots is provided for illustration.

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To realize the theoretical potential of quantum computation [1–3], it is essential to confront the difficult task of designing and constructing a functioning quantum computer [4]. An impressive body of quantum error correction literature has shown how, given a limitless supply of qubits and gates with fixed (small) decoherence, the qubits can be connected to execute quantum computation algorithms of arbitrary complexity [5–7]. The daunting problem of supplying the qubits and gates, however, remains unsolved. After an intensive effort to find physical implementations [8–21], it remains unclear whether decoherence can be reduced enough to make a useful quantum computer. In this paper, instead of a specific implementation, we suggest an approach to the problem that inherently defends against decoherence with an energy barrier [22]. This is achieved by proposing that a qubit be constructed not as a two-state quantum system developing through N unitary time evolutions but, instead, as a time-independent quantum system developing through a $2(N+1)$-dimensional Hilbert space.

To formulate this time-independent approach to quantum computation, let us first review the usual time-dependent approach. Suppose that a quantum algorithm requires the wave function of a qubit to develop through N time evolutions U_j, $j=1,\ldots,N$. Here, each U_j is a two-by-two unitary matrix. To be concrete, suppose that the qubit is realized as a single electron that can occupy a localized state on a left quantum dot or a localized state on a right quantum dot (Fig. 1). Although we fully appreciate that such an implementation may be experimentally impractical, in principle it makes a sensible qubit, and it is convenient for illustration. The U_j coherently shift the wave function of the electron back and forth between the two localized states in accordance with

$$|\psi(t_i)\rangle = U_i|\psi(t_{i-1})\rangle \tag{1}$$

as depicted in Fig. 2. Since the wave function consists of two amplitudes at each time, and there are $(N+1)$ times, the progress of the algorithm can be described with $2(N+1)$ amplitudes.

In our time-independent approach, a qubit would not be realized as a single electron shared between two dots but, instead, as a single electron shared between $2(N+1)$ dots (Fig. 3). The amplitudes giving the electron's wave function on each dot would supply the $2(N+1)$ amplitudes needed to describe the progress of the algorithm. To show precisely how this works, suppose that $c^\dagger_{i,0}|\text{vac}\rangle$ and $c^\dagger_{i,1}|\text{vac}\rangle$ are states localized on the left and right dots of row i, respectively. The wave function of the electron, $|\Psi\rangle$, would be a superposition of such states. If we group together the creation operators into $C^\dagger_i=[c^\dagger_{i,0}c^\dagger_{i,1}]$, it follows that the operator $P_i=C^\dagger_i C_i$ projects onto the two-dimensional subspace of states at row i. Then, by analogy with Eq. (1), we require that

$$P_i|\Psi\rangle = U_i A_{i,i-1} P_{i-1}|\Psi\rangle. \tag{2}$$

Here, $A_{i,i-1}=C^\dagger_i C_{i-1}$ just moves the electron from row $i-1$ to row i. In words, Eq. (2) states that the wave function at each row is related to the wave function at the previous row by a specified unitary transformation.

If the electron wave function $|\Psi\rangle$ satisfies Eq. (2), then it can be interpreted as a development according to the algorithm, from the input state $P_0|\Psi\rangle$ at row 0 to the output state $P_N|\Psi\rangle$ at row N. However, how can we force the electron to satisfy Eq. (2)? This is achieved by constructing the Hamiltonian in the $2(N+1)$ space such that the ground-state wave function of the system satisfies Eq. (2).

A particularly convenient Hamiltonian is $H=\Sigma_{i=1}^N h^i(U_i)$, where

$$h^i(U) \equiv \epsilon[C^\dagger_{i-1}C_{i-1} + C^\dagger_i C_i - (C^\dagger_i U C_{i-1} + \text{H.c.})] \tag{3}$$

and the constant energy ϵ defines the energy scale of the Hamiltonian. This Hamiltonian is positive semidefinite and has two degenerate ground states of zero energy. The two ground states both satisfy Eq. (2), but one has $P_0|\Psi\rangle = c^\dagger_{0,0}|\text{vac}\rangle$ and the other has $P_0|\Psi\rangle = c^\dagger_{0,1}|\text{vac}\rangle$. The two possibilities correspond to different possible input values. The complete Hamiltonian for a calculation with a specific

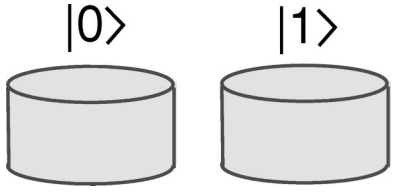

FIG. 1. Electron shared between two quantum dots constitutes a hypothetical qubit.

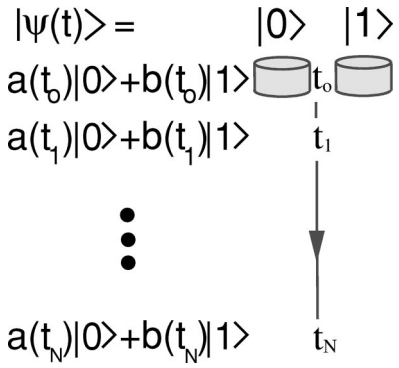

FIG. 2. Quantum dot qubit develops in time in accordance with the algorithm.

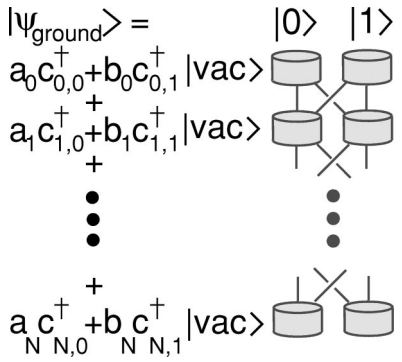

FIG. 3. Electron in an array of quantum dots forms a single qubit. Amplitude of wave function develops through array in accordance with the algorithm. Lines indicate tunneling paths.

input value does not possess this degeneracy, but rather includes a small perturbation term so that the energies of $c_{0,0}^{\dagger}|\mathrm{vac}\rangle$ and $c_{0,1}^{\dagger}|\mathrm{vac}\rangle$ differ slightly. As a result, there is a single nondegenerate ground state.

Physically, the Hamiltonian (3) would be realized in the quantum dot system by fabricating the dots such that they possess appropriate on-site and tunneling matrix elements. The four values in the matrix U_i would determine the four tunneling matrix elements that connect the states in row $i-1$ to the states in row i. The four values of U_i would not be required to influence any other aspect of the array since the operator (3) has an appealing modular character: the unitary matrix U_i only enters the Hamiltonian through matrix elements between states on rows $i-1$ and i. The small perturbation added to break the degeneracy of (3) and select input would be supplied physically by applying a voltage to one of the dots in row 0. After application of the perturbation, the system would be annealed to its ground state. The output of the calculation would be obtained by measuring on which of the dots at row N the electron can be found.

What are the advantages of this ground-state approach to quantum computation? Most importantly, it possesses a certain robustness against decoherence. Certainly, time-dependent or time-independent perturbations of the Hamiltonian could introduce errors into the calculation. Static perturbations due to imperfect implementation of the requisite Hamiltonian will adversely influence the ground state. If a ground-state quantum computer is to function, such time-independent sources of decoherence must first be removed by testing and refining the computer apparatus. (Thus ground-state quantum computation does not require time-dependent control of a system, but it does demand fine tunability of a static Hamiltonian. The required precision of implementation is as high as it is in the case of time-dependent quantum computation.) However, the inevitable, uncontrollable time-dependent perturbations from the environment only influence the calculation if they excite the system out of the ground state. In a traditional quantum computation these fluctuations lead to decoherence. In ground-state quantum computation, such excitations can be quenched by large energy level spacings and low temperatures. While the energy spacing of the Hamiltonian (3) does decrease with N, it can be proven that the decrease is only algebraic and, in the multiple qubit case to be addressed below, need not decrease as more qubits are added [23].

A second advantage to the ground-state approach is the fact that the system does not require time-dependent control. The Hamiltonian is static during any given calculation and only changes when switching from one algorithm to another. (For the quantum dot example that we give, this alteration could be implemented by using intermediate dots of adjustable voltage to control interdot tunneling rates.) This could ease somewhat the problem of realizing a quantum computer.

There are two disadvantages to our ground-state approach that should be pointed out. First, rather than having a small system that evolves in time through N time steps before decoherence sets in, we require $N+1$ copies of the small system. This could be an inefficient use of hardware. (Although it must be noted that a small time-dependent system can require a tremendous amount of hardware to control its evolution.) Second, to detect the results of a calculation, an electron must be measured in row N. Since this does not occur with certainty, there is a chance that the system will have to be annealed to the ground state again after an unsuccessful measurement. However, it is possible to reduce this problem drastically by adjusting $h^N(U)$ in the Hamiltonian (3) to read

$$h^N(U) \equiv \epsilon \left[C_{N-1}^{\dagger} C_{N-1} + \frac{1}{\lambda^2} C_N^{\dagger} C_N - \frac{1}{\lambda} (C_N^{\dagger} U C_{N-1} + \mathrm{H.c.}) \right]. \tag{4}$$

This will change the ground state so that, while it still satisfies Eq. (2), the probability that the electron is at the last row is enhanced by a factor of $\lambda^2/[\lambda^2+(N-1)]$. For $\lambda \sim N$, the probability of an unsuccessful measurement becomes small.

Up to this point, we have focused upon a single qubit. To perform useful quantum computations, we must consider the M qubit case. It is natural to redefine $|\Psi\rangle$ to be an M qubit state, $\mathcal{U}_j$ to be a 2^M by 2^M unitary operator specified by the algorithm, $P_i \equiv \Pi_a C_{a,i}^{\dagger} C_{a,i}$ to be a multiple qubit projector, and $A_{i,i-1} \equiv \Pi_a C_{a,i}^{\dagger} C_{a,i-1}$ to be a multiple qubit mover, where the index a specifies the qubit upon which a given operator acts. Unfortunately, if we simply insert these redefinitions into Eq. (2), the result constrains but does not fully specify the M qubit wave function. It contains no informa-

tion about those terms in the many body wave function that are annihilated by every projector P_i, i.e., terms in which not all electrons are at the same row of the computer. We must posit a development equation that will specify all these terms (which have no analog in time-dependent quantum computation), but will result in a $|\Psi\rangle$ that is the ground state of a simple Hamiltonian. This is achieved by defining $|\Psi^j\rangle$ to be the ground state of a hypothetical computer, with only j of the actual computer's N rows, and requiring

$$|\Psi^j\rangle = \prod_{a=1}^{M} (1 + C^\dagger_{a,j} U_{a,j} C_{a,j-1}) |\Psi^{j-1}\rangle. \tag{5}$$

It is straightforward to check that this more specific equation implies the multiple qubit redefinition of Eq. (2) and the appropriate Hamiltonian is just the sum of the familiar one qubit Hamiltonians (3).

With Eq. (5), we are now in a position to include the essential two qubit controlled-NOT gate. Assume the algorithm specifies as the jth operation $\mathcal{U}_j$ a controlled-NOT of qubit B by qubit A and unitary operations $U_{a,j}$ on the other qubits $a \neq A,B$. The desired multiple qubit redefinition of Eq. (2) will still hold if we modify Eq. (5) at row j to read

$$\begin{aligned} |\Psi^j\rangle = & [1 + c^\dagger_{A,j,0} c_{A,j-1,0} (1 + C^\dagger_{B,j} C_{B,j-1}) \\ & + c^\dagger_{A,j,1} c_{A,j-1,1} (1 + C^\dagger_{B,j} N C_{B,j-1})] \\ & \times \prod_{a \neq A,B} (1 + C^\dagger_{a,j} U_{a,j} C_{a,j-1}) |\Psi^{j-1}\rangle, \end{aligned} \tag{6}$$

where N is the two-by-two NOT matrix (the Pauli matrix σ_x). The actual ground state will satisfy this requirement if the Hamiltonian is a sum of one-qubit Hamiltonians (3) and a two-body term of the form

$$\begin{aligned} h^j_{A,B}(CNOT) = & \epsilon C^\dagger_{A,j-1} C_{A,j-1} C^\dagger_{B,j} C_{B,j} + h^j_A(I) C^\dagger_{B,j-1} C_{B,j-1} \\ & + c^\dagger_{A,j,0} c_{A,j,0} h^j_B(I) + c^\dagger_{A,j,1} c_{A,j,1} h^j_B(N). \end{aligned} \tag{7}$$

Physically, in the quantum dot realization, the terms in Eq. (7) correspond to having the location of one qubit influence (by the Coulomb interaction) the tunneling matrix elements of another qubit.

We note in conclusion that the idea of using a time-independent Hamiltonian for quantum computation has been raised before [24,25]. This prior work, however, aimed to perform the usual time-dependent quantum computation using a static ''cursor Hamiltonian.'' Such an approach (i) requires tailoring a Hamiltonian with specific three-particle interactions and (ii) demands time-dependent state preparation and measurement. In addition, it (iii) is particularly susceptible to decoherence in the form of unintended reflections [26] that are not relevant to our design. The ''cursor Hamiltonian'' is probably, therefore, unfeasible. Our approach does not suffer from these basic problems, but its ultimate viability can only be assessed by developing specific implementations. Our quantum dot array implemention seems unrealistic (although it is encouraging to note that a classical computation scheme using coupled quantum dots has been implemented [27,28]). However, many other possibilities could be envisioned, e.g., the $2(N+1)$ states of a single qubit could take different locations in momentum space rather than different locations in real space. Perhaps it will even turn out to be fruitful to combine the approach we describe here with other ways of handling decoherence [5–7,29].

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